AUSTRALIAN MATHEMATICAL OLYMPIAD COMMITTEE

2010 IMO Team Training

Exam T16

- Each question is worth 7 points.
- Time allowed is $4\frac{1}{2}$ hours.
- No books, notes or calculators permitted.
- Any questions must be submitted in writing within the first half hour of the exam.

2010 Mathematical Ashes

1. Find the largest possible integer k, such that the following statement is true:

   "Let 2009 arbitrary non-degenerate triangles be given. In every triangle the three sides are coloured, such that one is blue, one is red and one is white. Now, for every colour separately, let us sort the lengths of the sides. We obtain

   $$\begin{array}{rll} & b_1 \leq b_2 \leq \ldots \leq b_{2009} & \text{the lengths of the blue sides,} \\ & r_1 \leq r_2 \leq \ldots \leq r_{2009} & \text{the lengths of the red sides,} \\ \text{and} & w_1 \leq w_2 \leq \ldots \leq w_{2009} & \text{the lengths of the white sides.} \end{array}$$

   Then there exists k indices j such that we can form a non-degenerate triangle with side lengths b_j, r_j, w_j."

2. Given a cyclic quadrilateral $ABCD$, let the diagonals AC and BD meet at E and the lines AD and BC meet at F. The midpoints of AB and CD are G and H, respectively.

   Show that EF is tangent at E to the circle through the points E, G and H.

3. Let $P(x)$ be a non-constant polynomial with integer coefficients. Prove that there is no function T from the set of integers into the set of integers such that the number of integers x with $T^n(x) = x$ is equal to $P(n)$ for every $n \geq 1$, where T^n denotes the n-fold application of T.