NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

British Mathematical Olympiad

Tuesday 4th March 1986

Time allowed - $3\frac{1}{2}$ hours.

Write on one side of the paper only. Start each question on a fresh sheet of paper. Arrange your answers in order. Put your full name, age (in years and months) and school on the top sheet of your answers. On each other sheet put your name and initials.

1. Reduce the fraction $\frac{N}{D}$ to its lowest terms when

$$N = 2244851485148514627,$$
$$D = 8118811881188118000.$$

2. A circle S of radius R has two parallel tangents t_1, t_2. A circle S_1 of radius r_1 touches S and t_1; a circle S_2 of radius r_2 touches S and t_2; also S_1 touches S_2 and all the circle contacts are external. Calculate R in terms of r_1 and r_2.

3. Prove that if m, n, r are positive integers and

$$1 + m + n\sqrt{3} = (2 + \sqrt{3})^{2r-1}$$

then m is a perfect square.

4. Find, with proof, the largest real number K (independent of a,b,c) such that the inequality

$$a^2 + b^2 + c^2 > K(a+b+c)^2$$

holds for the lengths a, b, c of the sides of any obtuse-angled triangle.

5. Find, with proof, the number of permutations

$$a_1, a_2, \ldots, a_n$$

of $1, 2, \ldots, n$ such that

$$a_r < a_{r+2} \quad \text{for} \quad 1 \leqslant r \leqslant n-2$$

and

$$a_r < a_{r+3} \quad \text{for} \quad 1 \leqslant r \leqslant n-3.$$

(In a permutation each of the numbers $1,2,\ldots,n$ appears.)

6. AB, AC, AD are three edges of a cube. AC is produced to E so that AE = 2AC and AD is produced to F so that AF = 3AD. Prove that the area of the section of the cube by any plane parallel to BCD is equal to the area of the section of tetrahedron ABEF by the same plane.